ADLV & ALV

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<u>Setups for ADLV</u>

F nonarch field, G conn red gp$/F$.

σ Frob of $\breve{F}/F$. $\breve{G} = G(\breve{F}) \circlearrowleft \sigma$.

$\breve{G} = \coprod_{w \in \tilde{W}} IwI$ ($\leftrightarrow$ KR strata) $\qquad$ I σ-stable Iwahori subgp.

$= \coprod_{\mu \in X_*(T)^+} K\mu K$ Cartan decomp ($\leftrightarrow$ EO strata)

(when G is quasi-split).

$= \coprod_{[b] \in B(G)_\sigma} [b] \leftarrow \sigma$-conj class of $\breve{G}$ ($\leftrightarrow$ Newton strata)

<u>Motivation</u> Understand the intersections

$$[b] \cap IwI \text{ and } [b] \cap K\mu K.$$

<u>Need</u> Group-theoretic model for the stratifications of the special fibers of reduction of Sh vars.

<u>Def'n</u> (Rapoport) For $b \in \breve{G}$, $w \in \tilde{W}$.

- ADLV in affine flag

$$X_w(b) = \{gI \in \underset{\text{affine flag}}{\underbrace{\breve{G}/I}} : g^{-1} b \sigma(g) \in IwI\}$$

- ADLV in affine Gr

$$X_\mu(b) = \{gK \in \breve{G}/K : g^{-1} b \sigma(g) \in K\mu K\}$$

<u>Fact</u> $X_w(b)$ & $X_\mu(b)$ are either empty or of fin dim.

<u>Fundamental questions</u> (1) Nonemptyness, (2) Dimension

(3) Conn comps, (4) Irred comps, (5) More explicit generic str.

Setups for ALV

$L = \breve{F}$ (if F exists) or $\mathbb{C}((\varepsilon))$, G conn red gp/L. $G = G(L)$

$\forall \gamma \in \breve{G}$ denote by $\{\gamma\}$ ordinary conj class of $\breve{G}$.

$$\breve{G} = \coprod_{w \in \tilde{W}} IwI \qquad I \text{ Iwahori subgp.}$$

$$= \coprod_{\mu \in X_*(T)^+} K\mu K \quad \text{Cartan decomp} \quad (\text{when } G \text{ is quasi-split}).$$

$$= \coprod_{\gamma} \{\gamma\}$$

Q. How to understand the intersections $\{\gamma\} \cap IwI$ & $\{\gamma\} \cap KwK$?

Def'n (Lusztig, Kottwitz-Viehmann) For $\gamma \in \breve{G}$, $w \in \tilde{W}$.

- ALV in affine flag

$$Y_w(\gamma) = \{gI \in \underset{\substack{\uparrow \\ \text{affine flag}}}{\breve{G}/I} : g^{-1}\gamma g \in IwI\}.$$

- ALV in affine Gr

$$Y_\mu(\gamma) = \{gK \in \breve{G}/K : g^{-1}\gamma g \in K\mu K\}.$$

Will prove $Y_w(\gamma)$, $Y_\mu(\gamma)$ are either ϕ or fin-dim'l when $\gamma \in \breve{G}^{rs}$ (regular semisimple).

Motivations (1) Encode information on the orbital integrals.

(2) (Conjecturally) theory of character sheaves on function fields analog of p-adic gps? (e.g. $GL_n(\mathbb{F}_q((s)))$.)

Rem ALV v.s. affine Springer fiber:

$$ASF = Y_1(\gamma) \text{ with } 1 = w \in \tilde{W} = Y_0(\gamma) \text{ with } 0 = \text{zero cowt} \in X_*(T)^+.$$

	ADLV	ALV
nonemptyness pattern	In Gr: Rapoport-Richartz, Kottwitz, ..., Gashi In Fl: Görtz-Haines-Kottwitz-Reuman, Görtz-He-Nie, He. Felix.	In Gr/Fl: for split gps, Kottwitz-Viehmann, Chi. (update: can do with general gps)
dimension formula	In Gr: Chai, Rapoport. Kottwitz, GHKR. X. Zhu, In Fl: GHKR, He. Felix.	In Gr/Fl: (for split gps, in equal char) w/ large residue char, Bouthier, Chi. Can drop this in recent works.

Why ALV is more complicated than ADLV?

$Y_1(\gamma)$ affine Springer fiber v.s. $X_1(b) \neq \phi$ $(\Leftrightarrow b \simeq 1)$.

discrete set in this case.

<u>Main Thm</u> (He) $\dim$ ALV $= \dim$ ADLV $+ \dim$ ASF.

$$\begin{array}{ccc} G/L & \xleftrightarrow{\text{matching}} & G'/F \quad (G' \text{ split}) \\ w & \longleftrightarrow & w \\ \{\gamma\} & \longmapsto & [b] \end{array}$$

$$\dim Y_w^G(\gamma) = \dim X_w^{G'}(b) + \dim \boxed{Y_\gamma^G}$$

affine Springer fiber of a Levi subgp of G determined by γ.

<u>Cor</u> In Gr, $Y_\mu(\gamma) \neq \phi \Leftrightarrow \kappa(\mu) = \kappa(\gamma)$ & $\nu_\gamma = \mu$.

"linear Newton slope"

In this case, $\dim Y_\mu(\gamma) = \langle \mu, \rho \rangle + \frac{1}{2}(d(\nu) - c(\nu))$

Here $d(\nu) =$ disc valuation of ν

$c(\nu) = \mathrm{rank}_L(G) - \mathrm{rank}_L(G_\nu)$.