Representations of $GL_n(D)$ near the identity

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A similar result holds true for R of any characteristic.

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2. Compute the dimension of the invariants of π by the j-th congruence subgroup of an arbitrary Moy-Prasad pro-p subgroup K when j is large enough

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The set $\mathfrak{O}(0)$ of nilpotent G-orbits in the Lie algebra $\mathfrak{g} = M_n(F)$ is in bijection with the set $\mathcal{P}(n)$ of partitions of n (well known).

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HC. extended only partially H. to any G. No integrality of the coefficients for a good choice of measures. No relation between the nilpotent integral orbitals in the RHS and the trace of a virtual finite length representation.

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There are measures such that $c_\pi(\mathfrak{O}) \in \mathbb{Q}$ for all $\mathfrak{O}$, using A. and generalizing the method of H. (Varma 2014).

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H-V. extends to the Grothendieck group $Gr_R(G)$ of admissible finite length representations and is compatible with Langlands-Jacquet LJ from $GL_{dn}(F)$ to $GL_n(D)$ when R is algebraically closed of characteristic $\neq p$ (Badulescu, Minguez-Sécherre).

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$$\supset K_1 = 1 + p_D \mathfrak{k}_0 \supset I_1 = 1 + p_D \mathfrak{i}_0 \supset I_{1/2+1} = 1 + p_D \mathfrak{i}_{1/2} \supset \ldots$$

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There exist an integer j_π and real positive numbers a_π, a'_π such that for $j \geq j_\pi$,

$$a_\pi p^{jd(\pi)} \leq \dim_R \pi^{K^{p^j}} \leq a'_\pi p^{jd(\pi)}$$

(Emerton-Paskunas 2020, $d(\pi)$ has been called the Gelfand-Kirillov dimension of π).

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When $[F : \mathbb{Q}_p] \leq 2$ they have finite length (Timmings 2023)