

From modular forms to Shimura varieties: an introduction

The purpose of this week-long program is to discuss basics of Shimura varieties, starting from more familiar concepts such as modular forms and modular curves, passing through their automorphic interpretations, to the concept of Shimura varieties. The program will consist of ten lectures, each accompanied with a set of exercises for discussion and for the students to better absorb the new material.

Dates: July 26–30, 2021.

Place: BICMR, Peking University; room 甲乙丙报告厅

A typical day of the program is as follows:

9:00–11:00	11:15–12:15	12:15–13:30	13:30–15:30	15:30–16:00	16:00–17:00
Lecture	Problem Session	Lunch	Lecture	Tea Break	Problem Session

A tentative plan for the ten lectures is as follows:

Lecture 1 Adelic interpretation of modular forms and automorphic representations

Lecture 2 Representations over local fields, nonarchimedean and archimedean

Lecture 3 Matsushima formula and moduli interpretation

Lecture 4 Geometric modular forms, Kodaira–Spencer isomorphism, Eichler–Shimura isomorphism

Lecture 5 Compactification of modular curves

Lecture 6 Galois representations associated to modular forms

Lecture 7 Siegel modular varieties, Shimura varieties of PEL type

Lecture 8 General theory of Shimura varieties

Lecture 9 Dual BGG complex and Hodge theory

Lecture 10 Cohomology of automorphic vector bundles and étale cohomology of Shimura varieties

Prerequisite: Modular forms, Algebraic Number Theory, and Algebraic Geometry.

(For étale cohomology, it does not hurt to view it as usual topological cohomology together with a Galois action.)