Elementary Number Theory

HINTS FOR PROBLEM SET 1

WENHAN DAI

Problem 1. (a) *Hint.* The best chance for a solution is to take a to be as small as possible and hope that the corresponding y is non-negative.

(b) *Hint.* In order for some non-negative integer x to be part of a solution, we need there to exist an integer y so that $n - ax = by$. This suggests considering the set $S = \{x \in \mathbb{Z}_{\geqslant 0} : b \mid n - ax\}$.

Problem 2. *Hint.* This screams induction. Induct on something that measures the "size of a and b". This question is basically asking you to prove that the Euclidean Algorithm works, using induction. If you prove it without induction, make sure your wordings are precise enough.

Problem 3. *Hint.* Define a convenient $f : \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{\geqslant 0} \to \mathbb{Z}_{\geqslant 0}$ and use Problem 2.

Problem 4. *Hint.* An integer n is a perfect square if and only if $v_p(n)$ is even for every prime p.

Problem 5. *Hint.* Prove $m \mid a_m$ first.

School of Mathematical Sciences, Peking University, 100871, Beijing, China
Email address: daiwenhan@pku.edu.cn