Ref: Le Bras, Espaces de Banach-Colmez et faisceaux cohérent sur la courbe de Fargues-Fontaine.

Goal of this talk:

1. Introduce the settings and define the category

$\mathcal{BC}$ of Banach-Colmez spaces [Le Bras, §2.1-2.2]

30 minute

2. Calculate $\mathrm{Ext}^i(A, B)$ [Le Bras, §4].

$A, B = \mathbb{G}_a$ or $\mathbb{Q}_p$.

1 hour.

(Warning: Highly detailed & messy!)

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§1. The category of Banach-Colmez spaces.

Settings

Fix $K = C$ or $K = C^b$.

Let Perf_K = cat. of perfectoid spaces$/K$

adic space covered by $\mathrm{Spa}(R, R^+)$
with R perfectoid K-algebra.

Def: X, Y: analytic adic spaces.
$f: X \to Y$ is called pro-étale
if analytic locally on X and Y,
we may write

$$f: Y = \varprojlim Y_i \longrightarrow X$$
$$Y = \mathrm{Spa}(S, S^+), \quad Y_i = \mathrm{Spa}(S_i, S_i^+), \quad X = \mathrm{Spa}(R, R^+)$$

s.t. each $\mathrm{Spa}(S_i, S_i^+) \to \mathrm{Spa}(R, R^+)$ is étale.

Def: (1). The site $\mathrm{Perf}_{K,\mathrm{proét}}$

= big pro-étale site $/K$

= Perf_K + coverings given by

$\{ f_i : S_i \xrightarrow{\text{pro-étale}} S, \ i \in I \}$ satisfying

$(*)$ for every qc open $U \subseteq S$, there exist

a finite set $J \subseteq I$ and qc opens $U_j \subseteq S_j$

$\forall j \in J$, s.t. $U = \bigcup_{j \in J} f_j(U_j)$

(2). Y: analytic adic space

The site $Y_{\text{proét}}$

= small pro-étale site of Y.

$$= \begin{cases} \text{object}: f: X \longrightarrow Y \text{ proétale} \\ \qquad\quad\ \ \rotatebox{90}{$\in$} \\ \qquad\quad \text{Perf}_K \\ \text{Covering}: \text{similar as in (1).} \end{cases}$$

(3). The site $\text{Perf}_{K,v}$

= v-site of K

= Perf_K + covering given by

$\{f_i : S_i \longrightarrow S, i \in I\}$ satisfying $(*)$

<u>Thm</u> (tilting equivalence) The tilting functor induces equivalences of sites:

$$\text{Perf}_{C,\text{proét}} \simeq \text{Perf}_{C^\flat,\text{proét}}, \quad \text{Perf}_{C,v} \simeq \text{Perf}_{C^\flat,v}.$$

Sheaves on $\mathrm{Perf}_{K,v}$.

Fact: (1). The presheaf

$$\begin{aligned} \mathbb{G}_a : \mathrm{Perf}_C &\longrightarrow Ab \\ X &\longmapsto \mathcal{O}_X(X) \end{aligned}$$

is a sheaf in $\mathrm{Perf}_{C,v}$ (thus also a sheaf in $\mathrm{Perf}_{C,\text{proét}}$)

(2). The sheaf $\mathbb{G}_a$ is representable by $\mathbb{A}^1_C$, the affine line$/C$ (not perfectoid)

when $C = C^\flat$, it is repr'ed by a perfectoid space $\mathbb{A}^{1,\mathrm{perf}}_C = \mathrm{Spa}(C^\flat\langle T^{1/p^\infty}\rangle)$

(3). T: top. space, the presheaf

$$\begin{aligned} \underline{T} : \mathrm{Perf}_C &\longrightarrow Ab \\ X &\longmapsto C^0(|X|, T) \end{aligned}$$

is a sheaf in v-topology.

(4). When T is profinite, the sheaf $\underline{T}$ is representable by a perfectoid space

$$X = \mathrm{Spa}(C^0(T, C), C^0(T, \mathcal{O}_C)).$$

continuous functions

Moreover, this perfectoid space is strictly totally disconnected, i.e. $\forall$ étale covering $U \longrightarrow X$ admits a section.

$(\Rightarrow H^i_{\acute{e}t}(X, \mathcal{F}) = 0 \quad \forall \mathcal{F} \in Ab(X_{\acute{e}t}), \forall i > 0.)$

Banach-Colmez spaces

Def: The category of Banach-Colmez spaces, denoted by $\mathcal{BC}$, is the smallest abelian subcategory of $Ab(Perf_{C,pro\acute{e}t})$ that is stable under extensions and containing $\mathbb{Q}_p$ and $\mathbb{G}_a$.

§ Calculation of extensions.

Thm: In the category $Ab(Perf_{C,pro\acute{e}t})$, we have

$$Ext^i(\mathbb{G}_a, \mathbb{G}_a) = \begin{cases} C & i = 0 \\ C & i = 1 \\ 0 & i = 2 \end{cases}$$

← in detail

$$\mathrm{Ext}^i(\mathbb{G}_a, \mathbb{Q}_p) = \begin{cases} 0 & i=0 \\ \mathbb{C} & i=1 \\ 0 & i=2 \end{cases}$$

in detail.

other 2 are similar

$$\mathrm{Ext}^i(\mathbb{Q}_p, \mathbb{G}_a) = \begin{cases} \mathbb{C} & i=0 \\ 0 & i=1,2 \end{cases}$$

$$\mathrm{Ext}^i(\mathbb{Q}_p, \mathbb{Q}_p) = \begin{cases} \mathbb{Q}_p & i=0 \\ 0 & i=1,2. \end{cases}$$

<u>Strategy:</u> For an abelian group G in topos T,

$\exists$ a canonical resolution $C^\bullet$ of G

$$\cdots \xrightarrow{\partial_4} \overbrace{\mathbb{Z}[G^4]\times\mathbb{Z}[G^3]\times\mathbb{Z}[G^3]\times\mathbb{Z}[G^2]\times\mathbb{Z}[G]}^{C^{-3}} \xrightarrow{\partial_3}$$

$$\underbrace{\mathbb{Z}[G^3]\times\mathbb{Z}[G^2]}_{C^{-2}} \xrightarrow{\partial_2} \underbrace{\mathbb{Z}[G^2]}_{C^{-1}} \xrightarrow{\partial_1} \underbrace{\mathbb{Z}[G]}_{C^0} \xrightarrow{\varepsilon} G.$$

with $\partial_1[x,y] = [x+y]-[x]-[y]$;

$\partial_2[x,y,z] = [x+y,z]-[y,z]-[x,y+z]+[x,y]$,

$\partial_2[x,y] = [x,y]-[y,x]$;

There is a spectral sequence

$$E_1^{p,q} = \mathrm{Ext}_T^{q}(C^{-p}, G') \Rightarrow \mathrm{Ext}_T^{p+q}(C^\bullet, G')$$

$$\mathrm{Ext}_T^{p+q}(C^\bullet, G') = \mathrm{Ext}_T^{p+q}(G, G')$$

And each $E_1^{p,q}$ is a product of terms of the form

$$\mathrm{Ext}_T^{i}(\mathbb{Z}[G], G')$$

Upshot: In our case, G is represented by some X (e.g. $G = \mathbb{G}_a$, $X = \mathbb{A}^1_C$)

$\mathrm{Ext}_T^i(\mathbb{Z}[G], G')$ equals to $H^i_{\mathrm{pre}}(X, G')$.

$$H^i_{\mathrm{pre}}(X, G') \overset{\text{denote}}{=} H^i(X, G')$$

Rmk:

Roughly speaking, $\forall X$ analytic adic space $/C$, $F \in Ab(X_{proét})$

$\mu : X_v \to X_{proét}$ induces

$$H^i_v(X, \mu^* F) = H^i_{proét}(X, F) \quad \forall i \geq 0$$

So these calculation of extensions also applies to v-topology.

The case $G = G' = \mathbb{G}_a$.

Input: (will be proved in next talk)

$$H^i(\mathbb{A}^n_C, \mathbb{G}_a) = \Omega^i(\mathbb{A}^n_C) \quad \forall n \geq 1,\ i \geq 0.$$

E_1-page $d : (p,q) \to (p+1, q)$

q

2: $\underset{\overset{\|}{0}}{H^2(\mathbb{A}^1_C, \mathbb{G}_a)} \xrightarrow{d_1} H^2(\mathbb{A}^2_C, \mathbb{G}_a)$

1: $H^1(\mathbb{A}^1_C, \mathbb{G}_a) \xrightarrow{d_1} H^1(\mathbb{A}^2_C, \mathbb{G}_a) \xrightarrow{d_2} H^1(\mathbb{A}^3_C, \mathbb{G}_a) \times H^1(\mathbb{A}^2_C, \mathbb{G}_a)$

$\ker d_1 = C$ exact

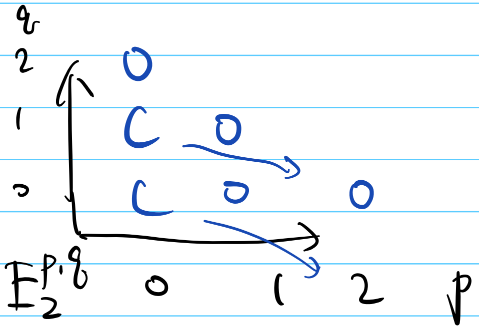
0: $H^0(\mathbb{A}^1_C, \mathbb{G}_a) \xrightarrow{d_1} H^0(\mathbb{A}^2_C, \mathbb{G}_a) \xrightarrow{d_3} H^0(\mathbb{A}^3_C, \mathbb{G}_a) \times H^0(\mathbb{A}^2_C, \mathbb{G}_a) \xrightarrow{d_3} H^0(\mathbb{A}^4_C, \mathbb{G}_a) \times H^0(\mathbb{A}^3_C, \mathbb{G}_a) \times$
$H^0(\mathbb{A}^3_C, \mathbb{G}_a) \times H^0(\mathbb{A}^2_C, \mathbb{G}_a) \times H^0(\mathbb{A}^1_C, \mathbb{G}_a)$

$\ker d_1 = C$ exact   exact

$E_1^{p,q}$ 0   1   2   3 p

E_2-page

$d: (p,q) \to (p+2, q-1)$



Single calculations

① $E_1^{0,2} = \Omega^2(\mathbb{A}^1_{\mathbb{C}}) = 0.$

②
$$\begin{array}{ccc} E_1^{0,1} & \xrightarrow{\partial_1} & E_1^{1,1} \\ \parallel & & \parallel \\ \Omega^1(\mathbb{A}^1_{\mathbb{C}}) & \longrightarrow & \Omega^1(\mathbb{A}^2_{\mathbb{C}}) \\ \omega(x) & \longmapsto & \omega(x+y) - \omega(x) - \omega(y) \end{array}$$

$\ker \partial_1 = \{\omega \mid \omega = c\, dx\} \cong \mathbb{C}.$

③
$$\begin{array}{ccc} E_1^{0,0} & \xrightarrow{\partial_1} & E_1^{0,1} \\ \parallel & & \parallel \\ \mathcal{O}(\mathbb{A}^1_{\mathbb{C}}) & \longrightarrow & \mathcal{O}(\mathbb{A}^2_{\mathbb{C}}) \\ f(x) & \longmapsto & f(x+y) - f(x) - f(y) \end{array}$$

$\ker \partial_1 = \{ f \mid f(x) = c x,\ c \in \mathbb{C}\}$
$\cong \mathbb{C}.$

Complicated calculations:

④ exactness at $E_1^{1,0}$

$$\begin{array}{ccccc} E_1^{0,0} & \xrightarrow{\partial_1} & E_1^{1,0} & \xrightarrow{\partial_2} & E_1^{2,0} \\ \| & & \| & & \| \\ \mathcal{O}(\mathbb{A}_C^1) & \xrightarrow{\partial_1} & \mathcal{O}(\mathbb{A}_C^2) & \xrightarrow{\partial_2} & \mathcal{O}(\mathbb{A}_C^3) \times \mathcal{O}(\mathbb{A}_C^2) \end{array}$$

$h(x) \longmapsto h(x+y) - h(x) - h(y)$

$f(x,y) \longmapsto (\partial_2^1, \partial_2^2)$

$\partial_2^1 = f(x+y, z) - f(x, y+z) + f(x,y) - f(y,z)$,

$\partial_2^2 = f(x,y) - f(y,x)$

Suppose $f(x,y) = \sum_{j,k \geq 0} a_{j,k} x^j y^k$

$h(x) = \sum_{j \geq 0} c_j x^j$

Then $f \in \mathrm{Im}(\partial_1)$

$\Longleftrightarrow \exists\, c_j \in \mathbb{C}$ s.t. $h(x) = \sum c_j x^j \in \mathcal{O}(\mathbb{A}_C^1)$

and

(a) ($x^j y^k$'s coefficient, $j, k > 0$)

$$a_{j,k} = \binom{j+k}{j} C_{j+k}$$

(b) ($x^0 y^k$, $k > 0$)

$$a_{0,k} = C_k - C_k = 0$$

(c) ($x^j y^0$, $j > 0$)

$$a_{j,0} = 0$$

(d) ($x^0 y^0$)

$$a_{0,0} = -C_0$$

Then $f \in \ker \partial_2 \iff$

(A). ($x^j y^k z^l$'s coefficient in $\partial_2' = 0$, $j, l > 0$, $k \geq 0$)

$$0 = \binom{j+k}{j} a_{j+k,l} - \binom{k+l}{k} a_{j,k+l}$$

(B). ($x^0 y^k z^l$, $l > 0$, $k \geq 0$)

$$0 = \cancel{a_{k,l}} - \binom{k+l}{k} a_{0,k+l} + \cancel{a_{k+l}}$$

$\iff$

$$a_{0,k+l} = 0.$$

(C). ($x^j y^k z^0$, $j > 0$, $k \geq 0$)

$$a_{j+k,0} = 0.$$

(D). ($x^0 y^k z^0$, $k \geq 0$)

$$0 = 0.$$

(E). ($x^j y^k$'s coefficient in $\partial_2^2 = 0$, $\forall j, k \geq 0$)

$$a_{j,k} - a_{k,j} = 0.$$

Now $\forall f \in \ker(d_2)$ just set

$$\begin{cases} c_n = \frac{1}{n} a_{n-1,1}, & n > 0 \\ c_0 = -a_{0,0} \end{cases}$$

Then (d) holds by construction.

(B), (C) $\Rightarrow$ (b), (c) holds.

(A) $\underset{(k=1)}{\Rightarrow}$ $(j+1) \cdot a_{j+1,l} = (l+1) a_{j,l+1}$. $(*)$

By induction on l, we show (a) i.e.

$$a_{j,l} = \binom{j+l}{j} c_{j+l} \quad \forall j > 0$$

when $l=1$. $a_{j,1} = \binom{j+1}{1} c_{j+1}$ be construction.

when $a_{j,l} = \binom{j+l}{j} c_{j+l}$, $\forall j > 0$.

$$a_{j,l+1} \overset{(*)}{=} \frac{j+1}{l+1} \cdot a_{j+1,l} = \frac{j+1}{l+1} \binom{j+1+l}{j+1} \cdot c_{j+1+l}$$

$$= \binom{j+l+1}{j} \cdot c_{j+l+1}.$$

$\Rightarrow$ (a) holds.

We see that

$\ker(\partial_2) = \operatorname{im}(\partial_1)$, i.e. exact at $E_1^{0,1}$

⑤ exactness at $E_1^{2,0}$

$$E_1^{1,0} \xrightarrow{\partial_2} E_1^{2,0} \xrightarrow{\partial_3} E_1^{3,0}$$

$$\begin{array}{ccccc} E_1^{1,0} & \xrightarrow{\partial_2} & E_1^{2,0} & \xrightarrow{\partial_3} & E_1^{3,0} \\ \| & & \| & & \| \\ \mathcal{O}(\mathbb{A}_c^2) & \xrightarrow{\partial_2} & \mathcal{O}(\mathbb{A}_c^3)\times\mathcal{O}(\mathbb{A}_c^2) & \xrightarrow{\partial_3} & \mathcal{O}(\mathbb{A}_c^4)\times\mathcal{O}(\mathbb{A}_c^3)\times\mathcal{O}(\mathbb{A}_c^3)\times\mathcal{O}(\mathbb{A}_c^2)\times\mathcal{O}(\mathbb{A}_c^1) \\ h & \longmapsto & (\cdots \quad \cdots) & & \\ & & (f, g) & \longmapsto & (\cdots \cdots) \end{array}$$

⑥ exactness at $E_1^{1,1}$

$$\begin{array}{ccccc} E_1^{0,1} & \xrightarrow{\partial_0} & E_1^{1,1} & \xrightarrow{\partial_1} & E_1^{2,1} \\ \| & & \| & & \| \\ \Omega^1(\mathbb{A}^1) & \xrightarrow{\partial_0} & \Omega^1(\mathbb{A}^2) & \xrightarrow{\partial_1} & \Omega^1(\mathbb{A}^3)\times\Omega^1(\mathbb{A}^2) \end{array}$$

similar explicit calculation

$\Rightarrow$ exactness at $E_1^{2,0}$, $E_1^{1,1}$

These terms in spectral sequence no longer change after E_2-page

$$\Rightarrow \operatorname{Ext}^i(\mathbb{G}_a, \mathbb{G}_a) = \begin{cases} \mathbb{C} & i=0 \\ \mathbb{C} & i=1 \\ 0 & i=2 \end{cases}$$

The case $G = \mathbb{G}_a$, $G' = \mathbb{Q}_p$. (sketch)

Input: (next week)

$H^0(\mathbb{A}^n_C, \mathbb{Q}_p) = \mathbb{Q}_p$

$H^1(\mathbb{A}^n_C, \mathbb{Q}_p) = \mathcal{O}(\mathbb{A}^n_C) / C$

$H^2(\mathbb{A}^1_C, \mathbb{Q}_p) = 0$. ← constant functions

Similiar analysis on E_1 page

$$\Rightarrow \mathrm{Ext}^i(\mathbb{G}_a, \mathbb{Q}_p) = \begin{cases} 0 & i=0 \\ C & i=1 \\ 0 & i=2 \end{cases}$$

Rmk: The fundamental exact sequence

$$0 \to \mathbb{Q}_p t \to B_{cris}^{+\varphi=p} \to \mathbb{G}_a \to 0$$

gives a nontrivial extension

in $\mathrm{Ext}^1(\mathbb{G}_a, \mathbb{Q}_p) = C$

The case $G = \mathbb{Q}_p$, $G' = \mathbb{G}_a$ on $\mathbb{Q}_p$

As $\mathrm{Hom}(\mathbb{Q}_p, -) = \varprojlim_{\times p} \mathrm{Hom}(\mathbb{Z}_p, -)$, we have SES

$$0 \to R^1\varprojlim_{\times p} \mathrm{Ext}^{i-1}(\mathbb{Z}_p, G') \to \mathrm{Ext}^i(\mathbb{Q}_p, G') \to \varprojlim_{\times p} \mathrm{Ext}^i(\mathbb{Q}_p, G') \to 0$$

so it suffice to show

$$\mathrm{Ext}^i(\mathbb{Z}_p, \mathbb{Q}_p) = \begin{cases} \mathbb{Q}_p & i = 0 \\ 0 & i > 0 \end{cases}$$

$$\mathrm{Ext}^i(\mathbb{Z}_p, C) = \begin{cases} C & i = 0 \\ 0 & i > 0. \end{cases}$$

<u>Input</u>: Let $X^n = \mathrm{Spa}(C^0(\mathbb{Z}_p^n, C), C^0(\mathbb{Z}_p^n, \mathcal{O}_C))$

X^n is perfectoid + strictly totally disconnected then (1) $H^i(X^n, \mathbb{G}_a) = \begin{cases} C^0(\mathbb{Z}_p^n, \mathbb{C}) & i=0 \\ 0 & i>0 \end{cases}$

(2) $H^i(X^n, \mathbb{Q}_p) = \begin{cases} C^0(\mathbb{Z}_p^n, \mathbb{Q}_p) & i=0 \\ 0 & i>0. \end{cases}$

Pf: $H^i(X^n, \mathbb{G}_a) = H^i(X^n, \hat{\mathcal{O}}_X)$

(1) just follows from X is affinoid perfectoid.

for (2), $H^i(X^n, \mathbb{Z}/p^k\mathbb{Z}) = H^i_{et}(X^n, \mathbb{Z}/p^k\mathbb{Z}) \underset{\Uparrow}{=} 0$

Strictly tot. discon.

We have the SES:

$0 \to R^1\varprojlim H^{i-1}(X^n, \mathbb{Z}/p^k\mathbb{Z}) \to H^i(X^n, \mathbb{Z}_p) \to \varprojlim H^i(X^n, \mathbb{Z}/p^k\mathbb{Z}) \to 0$

$\Rightarrow H^i(X^n, \mathbb{Z}_p) = 0$

and $H^i(X^n, \mathbb{Q}_p) = H^i(X^n, \mathbb{Z}_p)[\frac{1}{p}] = 0.$ $\square$

Now fix $G = \mathbb{Z}_p$, $G' = \mathbb{C}$. ($G' = \mathbb{Q}_p$ is similar)

E_1-page.

$$
\begin{array}{c|cccccccc}
q & & & & & & & \\
2 & 0 & \longrightarrow & 0 & & & & \\
1 & 0 & \longrightarrow & 0 & \longrightarrow & 0 & & \\
0 & E_1^{0,0} & \xrightarrow{\partial_1} & E_1^{1,0} & \xrightarrow{\partial_2} & E_1^{2,0} & \xrightarrow{\partial_3} & E_1^{3,0} \\
 & \| & & \| & & \| & & \| \\
 & C^0(\mathbb{Z}_p, \mathbb{C}) & & C^0(\mathbb{Z}_p^2, \mathbb{C}) & & C^0(\mathbb{Z}_p^3, \mathbb{C}) \times C^0(\mathbb{Z}_p^2, \mathbb{C}) & & C^0(\mathbb{Z}_p^4, \mathbb{C}) \times C^0(\mathbb{Z}_p^3, \mathbb{C}) \times C^0(\mathbb{Z}_p^3, \mathbb{C}) \times C^0(\mathbb{Z}_p^2, \mathbb{C}) \times C^0(\mathbb{Z}_p, \mathbb{C}) \\
\hline
E_1^{p,q} & 0 & & 1 & & 2 & & 3 \quad p
\end{array}
$$

<u>Claim</u>: (1). $\ker \partial_1 = \mathbb{C}$

(2). exact at $E_1^{1,0}$

(3). exact at $E_1^{2,0}$

(4). The spectral sequence no longer changes after E_2 page.

<u>sketch of (2).</u> :

<u>Fact</u>: (Mahler expansion)

the functions

$$(x,y) \longmapsto \binom{x}{k}\binom{y}{\ell} = \frac{x(x-1)\cdots(x-k+1)}{k!} \cdot \frac{y(y-1)\cdots(y-\ell+1)}{\ell!}$$

$\cap$

$\mathbb{Z}_p^2$

forms a basis of $C^0(\mathbb{Z}_p^2, \mathbb{C})$.

$\forall f \in C^0(\mathbb{Z}_p^2, \mathbb{C})$

we can write $f = \sum_{k,\ell} a_{k,\ell} \binom{x}{k}\binom{y}{\ell}$, with $a_{k,\ell} \to 0$

Then

$f \in \operatorname{im} \partial_2 \iff \exists h,\ f(x,y) = h(x+y) - h(x) - h(y)$

$\iff \exists c_j$ s.t. $\left(h(x) = \sum_{j \geq 0} c_j \binom{x}{j}\right)$

(a). $(x^j y^k,\ j,k > 0)$

$$a_{j,k} = c_{j+k}$$

(b). $(x^0 y^k,\ k > 0)$

$$a_{0,k} = 0$$

(c). $(x^j y^0,\ j > 0)$

$$a_{j,0} = 0$$

(d) $(x^0 y^0)$

$$a_{0,0} = -c_0$$

$f \in \ker \partial_2 \iff$

$$\begin{cases} f(x+y,z) - f(y,z) - f(x,y+z) + f(x,y) = 0 \\ f(x,y) - f(y,x) = 0 \end{cases}$$

Fact: $\binom{x+y}{k} = \sum_{j=0}^{k} \binom{x}{j}\binom{y}{k-j}$

$f \in \ker \partial_2$

$\Rightarrow$ (A) (coefficient of $x^j y^k z^l$ $\quad j, l > 0, k \geq 0$)

$$a_{j+k,l} = a_{j,k+l}.$$

(B) ($x^0 y^k z^l$, $l > 0$, $k \geq 0$)

$$\cancel{a_{k,l}} - \cancel{a_{k,l}} + a_{0,k+l} = 0$$

(C) ($x^j y^k z^0$, $j > 0$, $k \geq 0$)

$$a_{j+k,0} = 0$$

(D) $\quad a_{j,k} = a_{k,j}$

Now just choose $\quad c_j = a_{j-1,1} \quad \forall j > 1 \quad$ is OK.

$c_0 = -a_{0,0}$.

c_1 arbitrary

((B).(C) $\Rightarrow$ (b)(c). (A) $\Rightarrow$ (a) (d) is by construction.)

The claim shows

$$\mathrm{Ext}^i(\mathbb{Z}_p, \mathbb{C}) = \begin{cases} \mathbb{C} & i = 0 \\ 0 & i > 0 \end{cases}$$

$$\Rightarrow \mathrm{Ext}^i(\mathbb{Q}_p, \mathbb{C}) = \begin{cases} \mathbb{C} & i = 0 \\ 0 & i > 0 \end{cases} \qquad \square$$