Appendix.

A Grothendieck Topology on K-Aff is to assign each $X \in$ K-Aff a G-topology, s.t. for $Z \xrightarrow{\varphi} X$ a morphism,

$\varphi$ is continuous. the denote by $T_X / T$.

Def. Let $X \in \mathcal{C} \triangleq$ K-Aff, $U \subseteq X$, an arbitrary covering $\{U_i \to U\}$ is called compatible with $T$ if for any $Y \xrightarrow{\varphi} X$, and any $T_Y$-open $V \subseteq Y$,

s.t. $\varphi(V) \subseteq U$, the covering $\{\varphi^{-1}(U_i) \to V\}$ admit a refinement by $T_Y$-covering. (This is how weak $\longrightarrow$ strong)

Thm. For a G-topology $T$ on K-Aff, Define a topology $T'$:

① $U \subseteq X$ is open if it admits a cover $\{U_i \to U\}$, which is compatible with $T$.

② $\{V_j \to V\}$ a covering of $T'$-open is admissible if $\{V_j \to V\}$ is compatible with $T$.

Then $T'$ is the unique finest top among all the G-top. slightly finer than $T$.

$T'$ satisfies $(G_1)$-$(G_2)$ and $(G_0)$ if $T$ satisfies $(G_0)$.

Cor. Weak G-topology $\rightsquigarrow$ strong G-topology

strong G-topology satisfy $(G_0)$ — $(G_2)$ (Prop 5.2/5)

Lemma. (i) ~~If~~ $U, V$ are subsets of $X \in \mathcal{C}$, If $\{U_i \to U\}$ is ~~an ad~~ compatible with $T$, ~~then~~ $\{V \cap U_i \to V \cap U\}$ is compatible with $T$.

(ii) $\{U_i \to U\}$ is compatible, $\{U_{ij} \to U_i\}$ is compatible, ~~then~~ $\{U_{ij} \to U\}$ is compatible. $\square$

<u>proof</u> of $(G_1)$ $T$ for weak, $T'$ for strong.

$V \subseteq U$, $U \subseteq X$ admissible open, $\exists$ $(U_i \to U)$ admissible, st. $V \cap U_i$ is open in $X$, then $V$ is admissible open.

We need to construct a $T$-open ~~con~~ covering of $V$ compatible with $T$.

$(U_i \to U)$ compatible $\Rightarrow \{U_i \cap V \to V\}$ is compatible.

Now choose ~~$U_i \cap$~~ $\{V_{ij} \to U_i \cap V\}$ a covering making $U_i \cap V$ $T'$-open

Then $\{V_{ij} \to V\}$ is compatible with $T$ and $V_{ij}$ are $T$-open. $\square$